

The Origin of Density Fluctuations in a Cyclic Universe

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Abstract. This paper is a brief introduction to the “cyclic model of the universe,” a radical alternative to standard big bang/inflationary theory in which space and time exist indefinitely, inflation is avoided, and the universe undergoes periodic epochs of expansion and conclusion. This introduction explains the novel way in which density perturbations are generated which seed large scale structure formation and produce spatial variations in the cosmic microwave background temperature.

The cyclic model of the universe [1, 2] is a radical alternative to the standard inflationary/big bang model [3, 4]. One key difference is the general flow of the cosmic evolution. In the standard model, the universe expands from the big bang and proceeds monotonically, transforming the cosmos from hot to cold and from dense to dilute. In the cyclic model, the evolution is periodic. The universe undergoes epochs of expansion during which its temperature and density decrease, but, after the density becomes negligibly small, a sequence of events occurs which creates new matter and radiation that reheats the universe to high temperature and density and triggers a new period of expansion and cooling.

A second key difference is the events that shape the large scale structure of the universe. In the standard picture, a brief period of hyper-rapid expansion (inflation) shortly following the big bang makes the universe homogeneous and isotropic, flattens the spatial curvature, and creates a nearly scale-invariant spectrum of density fluctuations. In the cyclic model, inflation is replaced by physical processes that occurred a cycle ago, before the last bang. These processes entail energies and timescales that are one hundred orders of magnitude different from inflation. Furthermore, the physics that creates the density fluctuations that is fundamentally different.

Despite these extraordinary differences, the cyclic model appears capable of reproducing all of the successful predictions of the standard model with fewer ingredients. Not all predictions are identical, either. The cyclic and inflationary models have exponentially different predictions for the spectrum of primordial gravitational waves, for example. So, there are future tests which can discriminate the two models. At present, though, both models are in equally good agreement with the data.

THE CYCLIC MODEL: THE BRANE PICTURE

The cyclic model is inspired by recent concepts in superstring theory, particularly M-theory, the Horava-Witten model, branes, orbifolds and extra dimensions [6, 1], as well as a precursor cosmological model known as the “ekpyrotic scenario” [7]. The cosmological model does not *require* these features. We can pose the theory in a field theoretic language that is more easily compared to inflation. However, the M-theory description provides a simple and compelling geometrical picture that provides a natural intuition about how the model works. So, we will present here both the M-theory and field theory descriptions here.

In heterotic M-theory models, our three-dimensional universe is a hypersurface embedded in a spacetime with an extra spatial dimension. (Actually, in the Horava-Witten model, there are 6 additional spatial dimensions compactified on a Calabi-Yau manifold, but the manifold is so small that the six dimensions can be neglected for our purposes.) The hypersurface is a boundary or orbifold plane of the extra dimension separated by a finite distance from a second boundary/orbifold. The orbifolds have energy and momentum. They can interact through gravity and exchange virtual membranes. In the cyclic model, the orbifolds are drawn together by these interactions, and they collide and bounce at regular intervals.

The model goes through the following stages. Each cycle begins with a “bang,” a collision between branes that creates matter and radiation. The universe proceeds directly to the radiation-dominated epoch without encountering any inflation. The model must introduce a mechanism for making the universe homogeneous, isotropic and flat, and for creating a nearly scale-invariant spectrum of density perturbations. However, this will be accomplished by a sequence of events that occurs at a different point in the cycle. Hence, the universe proceeds directly after the bang to radiation domination to matter domination and, finally, to dark energy domination.

In the big bang/ inflationary model, dark energy comes as a complete surprise. It is not predicted or required. Rather, dark energy is added *ad hoc* to make the model consistent with the recent observations of cosmic acceleration [8, 9, 10].

In the cyclic model, dark energy moves to center stage as an essential element of the cyclic model. Its source is the potential energy associated with the interaction between branes. When the branes are far apart, the energy is presumed to be small and positive, acting as a form of quintessence that causes the branes to stretch at an accelerating rate, expanding by a factor of two every 15 billion years. Continued for 100 doublings or a trillion years, the dark energy thins out the matter and radiation in the universe to a point where the universe approaches a homogeneous vacuum. Furthermore, any warps or curvature in the branes are stretched out. Hence, two of the roles of inflation, making the universe homogeneous and flat, are replaced by dark energy in the cyclic model. Dark energy also is important in making the cyclic solution a classical attractor. That is, if the branes are kicked away from the ideal cyclic orbit, the period of dark energy domination causes the evolution to converge after a cycle or two to the ideal evolution.

After the matter and radiation have been thinned out, the universe begins a period of “contraction.” But, unlike earlier cyclic models discussed in the 1920’s and 1930’s [11] our three dimensions do not contract and the temperature and density do not diverge. Rather, the extra dimension between the orbifolds contracts as the two branes approach

one another and head towards collision. The contraction ends in a “crunch” at which matter and radiation are created. The two branes bounce apart, but now filled with the newly created hot matter-radiation whose density dominates the older, thinned out matter-radiation from the previous cycle. Due to gravity, the new matter and radiation causes the branes to begin to stretch again and damp the motion of branes. The universe has returned to the same state as it was after the last bang and the cycle begins anew.

During the contraction phase, the branes undergo quantum fluctuations that cause them to wrinkle. For simple, exponentially decreasing interbrane potentials, the wrinkles form a scale-invariant spectrum. As a result of the wrinkles, the branes collide, bounce and reheat at different times at different locations. The collision thereby imprints a scale invariant spectrum of spatial variations in the temperature on the branes after the collision.

THE CYCLIC MODEL: THE FIELD THEORETIC PICTURE

As we have noted, the cyclic story can be described in terms of an ordinary four-dimensional field theory, which can be obtained by taking the long wavelength limit of the brane picture [1, 2]. The distance between branes becomes a moduli (scalar) field ϕ . The interbrane interaction is replaced by a scalar field potential, $V(\phi)$. The different stages in the cyclic model in the brane picture are in one-to-one correspondence to the motion of the scalar field along the potential. See Fig. 1.

Then, the action S describing describing gravity, the scalar field ϕ , and the matter-radiation fluid is:

$$S = \int d^4x \sqrt{-g} \left(\frac{1}{16\pi G} \mathcal{R} - \frac{1}{2} (\partial\phi)^2 - V(\phi) + \beta^4(\phi) \rho_R \right), \quad (1)$$

where g is the determinant of the Friedmann-Robertson-Walker metric $g_{\mu\nu}$, G is Newton’s constant and \mathcal{R} is the Ricci scalar.

Particularly notable is the coupling $\beta(\phi)$ between ϕ and the matter-radiation (ρ_R) density. This coupling is crucial because it accounts for the fact that the temperature and density do not diverge at the crunch. If β were set to unity, the solution to the equation of motion would be $\rho_R \propto 1/a^4$, where a is the scale factor. Then, at the crunch where $a \rightarrow 0$, the density would diverge. (This describes the case of the older cyclic models.) However, the β factor has the property that $\beta \rightarrow \infty$ as $a \rightarrow 0$ such that $a\beta \rightarrow \text{constant}$. The revised solution to the equation of motion is $\rho_R \propto 1/(a\beta)^4$ which approaches a constant as $a \rightarrow 0$. The energy, once thinned out during the dark energy dominated phase, remains thinned out at the bounce.

If we had begun with the field theory and simply introduced the β -factor by hand, it would have seemed incredibly fine-tuned to choose a form which diverges as $1/a$. However, we now understand that the form arises automatically if ϕ is the modulus field that describes the size of the extra dimension. The β -factor simply reflects the fact that the extra-dimension collapses but our three-dimensions do not. As a result, entropy produced during one cycle is not concentrated at the crunch and does not contribute significantly to the entropy density at the beginning of the next cycle. Hence, cycles can

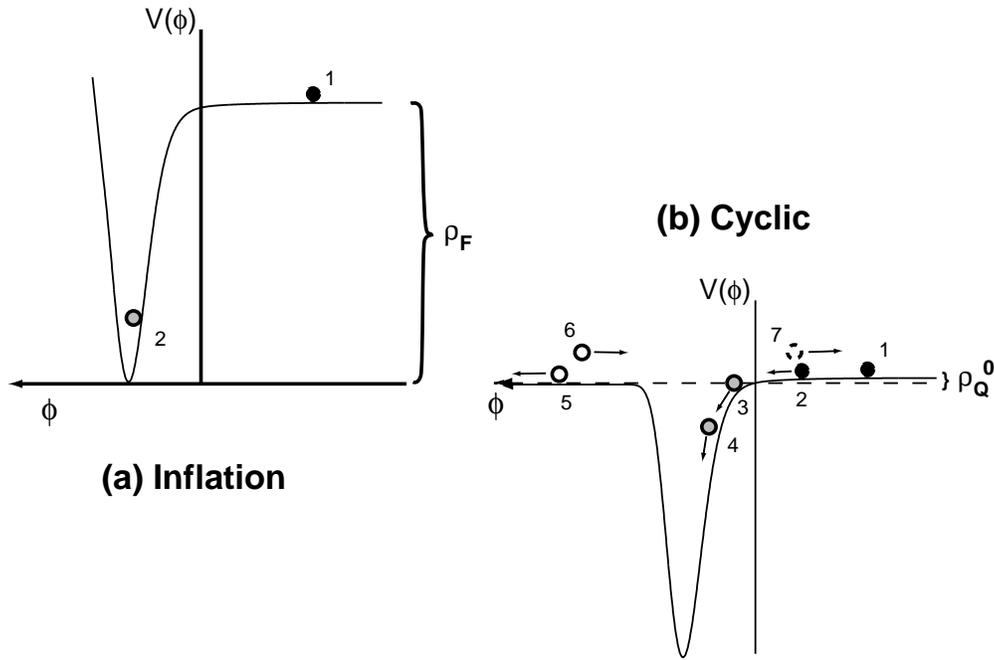


FIGURE 1. Schematic plot of the potential $V(\phi)$ as a function of the field ϕ for (a) inflationary cosmology and (b) cyclic models. For inflation, accelerated expansion and production of density perturbations occur in stage 1; reheating occurs at stage 2. For the cyclic model, present-day accelerated expansion occurs in stages 1 and 2; deceleration in stage 3; contraction and production of density perturbations begins in stage 4; contraction (dominated by scalar field kinetic energy) in stage 5; bounce, production of matter-radiation, and re-expansion in stage 6; matter-radiation dominated epoch begins in stage 7, and the cycle begins anew.

continue for an arbitrarily long time and there is no practical way of distinguishing one cycle from the next.

ORIGIN OF DENSITY PERTURBATIONS

If the cyclic model can be described in terms of ordinary field theory, then it may seem surprising that it is possible to generate a nearly scale invariant spectrum density perturbations. Perturbations in theories with scalar fields were investigated in the 1980's [12, 13], and it was found that a nearly scale-invariant arises if the scalar field has a nearly de Sitter equation of state. How does the cyclic model fit in? The perturbations cannot be generated during the dark energy dominated phase because the dark energy density is exponentially too small to generate a spectrum with the right amplitude.

The explanation, as has been recently discovered [14], is that there are actually three distinct ways of producing a nearly scale-invariant spectrum, and that inflation represents only one of them. The three ways can be characterized by $w \equiv (\frac{1}{2}\dot{\phi}^2 - V)/\frac{1}{2}\dot{\phi}^2 + V$, the effective equation of state of the scalar field. Case I is where $w \approx 1$ and the universe is expanding, the example of inflation. Case II is a contracting universe with $w \approx 0$,

an example which has not been used in cosmological models to date. Case III is a contracting universe with $w \gg 1$ – the situation that applies in the cyclic model. (Here I am only considering cases where w is nearly constant, cases which can be obtained with simple potentials; contrived examples can also be constructed in which w is time-dependent.)

We know how design a scalar field potential so that $w \approx -1$. If the potential is sufficiently flat (V'/V and V''/V are very small), then the field ϕ rolls slowly down the potential, V is nearly constant for an extended period, and w approaches -1 . It is in regime where the potential is flat that the perturbations are produced and, hence, where there are tight constraints on the form of $V(\phi)$. After the flat portion of the potential, there is a great deal of freedom in choosing the potential shape.

What is required to obtain $w \gg 1$? From the expression for w , it is apparent that this is only possible if the potential is negative. In particular, for a negative exponentially *steep* potential $V \approx -\exp(c\phi)$, the solutions to the equation of motion have a scaling solution in which $\dot{\phi}^2/2V$ is constant and approximately -1 . Consequently, w is much greater than unity and nearly constant. Curiously, a potential which rolls from positive to negative is just what is needed to go from an accelerating universe to a contracting universe, so the requirements for a scale-invariant spectrum dovetail with overall scenario. In analogy with inflation, this steep regime of the potential corresponding to the generation of fluctuations is where there are tight constraints on the form of the potential. Although the standard example of a potential has a flat positive plateau on one side and rise of V on the other [1, 2], there is actually tremendous flexibility in choosing the shape of $V(\phi)$ way from the steep portion [15]. See Fig. 2. For example, the positive plateau may be replaced by an increasing function or even a locally stable (positive energy) minimum. Similarly, the potential need approach zero or even have a minimum. (These features are put in the standard example motivated by M-theory [1, 16], but they are not required for cyclic cosmology.)

The generation of fluctuations for $w \gg 1$ can be understood *heuristically* by examining the perturbed Klein-Gordon equation [17]:

$$\delta\phi_k'' = - \left(k^2 + \frac{a''}{a} + V_{,\phi\phi} \right) \delta\phi_k \quad (2)$$

where $\phi(\mathbf{x}, t)$ has been expanded in fourier components $\delta\phi_k(t)$ with wavenumber k and prime is derivative with respect to conformal time η . The a''/a term is due to gravitational expansion, and the last term is due to the self-interaction of the scalar field. This equation applies equally to inflation and to cyclic models. The well-known result[] from inflation is that, in order to obtain a scale invariant spectrum, the combination of the last two terms on the right hand side must behave as $(2/\eta^2)\delta\phi_k$. In the case of inflation, $a(\eta) = -1/\eta$ and $V_{,\phi\phi}$ is negligible. So, the scale invariant fluctuations are due entirely to the gravity term. A second solution exists where $a''/a = 2/\eta^2$ and the gravity term dominates: namely, $a = \eta^2$, the dust-like $w = 0$ universe, Case II above.

The cyclic model corresponds to the limit where the gravity term is negligible and, instead, the perturbation equation is driven by the potential term. For the negative exponential potential, for example, the scaling solution corresponds to $V_{,\phi\phi} \approx 2/\eta^2$.

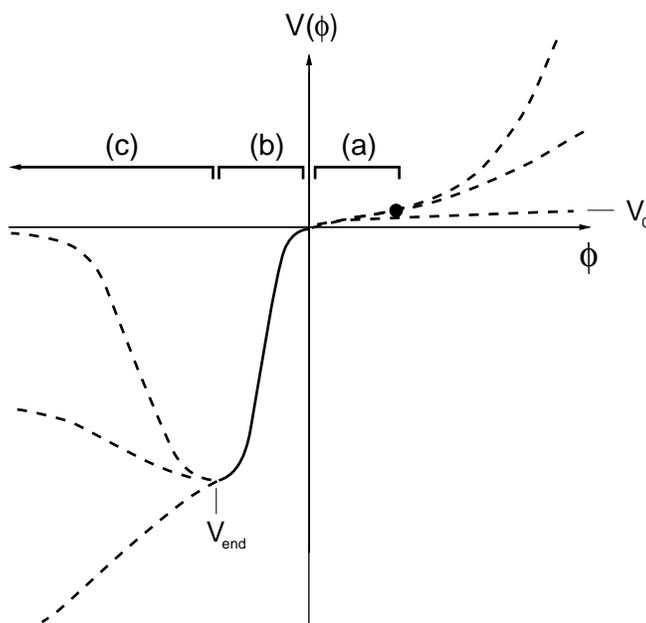


FIGURE 2. Plots of possible cyclic potentials showing how they can be viewed as having three separate parts: (a) positive potential energy density; (b) steep, negative potential; (c) less steep or increasing potential. The figure is intended to emphasize that the only tight constraints are for the negative, steep portion of the potential (b) where perturbations are generated.

In sum, we have seen a remarkable result: The perturbations are produced in inflationary models under conditions that are extremely different from cyclic models – hyper-rapid expansion as opposed to ultra-slow contraction – and yet the perturbation equations expressed in terms of η are isomorphic and the outcome is the same. Both produce a nearly scale-invariant, gaussian spectrum of adiabatic density perturbations.

We should note that the derivation of the density perturbation spectrum outlined here is not rigorous and it does not follow what happens to the perturbations at the bounce. At the bounce, fluctuations in the scalar field (or branes) lead to variations in the time of collision and of reheating to high temperature [1, 17]. In this way, the fluctuations are converted directly into a spectrum of density perturbations after the bounce. This argument is similar to the “time-delay” approach [13] to deriving perturbations in inflation.

Formal methods should use gauge invariant variables and include gravitational backreaction. The formal analysis reveals the same conclusions as the heuristic derivation above; the gravitational backreaction is negligible [17, 14]. Some other authors attempting their own methods have claimed the opposite result. A common error has been to choose a gauge invariant variable that does not include the growing mode perturbations to leading order. This mistake is easy to make since the appropriate gauge invariant variables for inflationary cosmology fail to include the growing mode perturbation in a contracting cyclic phase. Failure to take proper account of this difference results in accidentally projecting out the scale-invariant spectrum of fluctuations. A second common

error has been to ignore the radiation produced at the bounce. In our computation, this leads to elimination of the scale-invariant fluctuations [17]. The reason is clear: in the no-radiation limit, the evolution is time-reversal invariant. Any perturbations generated during contraction translate into purely decay perturbations after the bounce. The radiation breaks the symmetry between contracting and expanding. Now the scale-invariant perturbations produced during contraction match to a combination of growing and decaying modes after the bounce [17]. The growing mode provides the needed scale-invariant spectrum of density perturbations. However, this spectrum only exists if the bounce is not time-reversible.

GRAVITATIONAL WAVES

If both produce the same outcome, what is the difference? The outcome is only the same when it comes to density perturbations. The difference shows up in the spectrum of gravitational waves [1, 2, 17, 18]. To see this, consider that the perturbation equation above applies to all fields. For inflation, the gravity term dominates over the potential term, which is proportional to the mass of the inflaton during inflation. Gravity will, therefore, dominate in the equation describing any light mass fields, including the two massless polarization modes of the metric fluctuations. All such fields will have scale-invariant fluctuations. For most fields, this is irrelevant because the fluctuations leave no distinctive cosmic signature. Fluctuations of the massless metric perturbations, though, propagate as gravitational waves, leaving a distinctive signature in the cosmic microwave background anisotropy as well as in gravitational wave detectors. Hence, inflation predicts that there should be a nearly scale-invariant spectrum of gravitational waves in addition to density fluctuations.

For the cyclic model, the potential term dominates and the gravitational term is irrelevant. Hence, cyclic models only produce scale invariant fluctuations in the scalar field. For gravitational waves, there is only the a''/a term and the spectrum is very blue. That is, the amplitude drops from some small value at very short wavelengths to exponentially smaller values at cosmic scales.

This dramatic difference from inflationary cosmology (see Fig. 3 from [18]) is the most promising approach for distinguishing the two scenarios. In the near-term, measuring the polarization of the cosmic microwave background is most sensitive method for searching for stochastic gravitational waves. Gravitational waves leave a characteristic “B-mode” polarization pattern that is smaller in amplitude than the “E-mode” created by density perturbations, but distinctive [19, 20]. Detector sensitivities anticipated in the coming decade should be sufficient to cover a wide span of the most likely inflationary models [21]. As suggested in Fig. 3, direct detection of gravitational waves from inflation is not feasible in the next decade. Detection of the cosmic gravitational waves would clearly support inflation and disprove the cyclic scenario. The absence of detection would not disprove inflation, but it would force us towards more arcane inflationary models versus the comparatively simpler cyclic model.

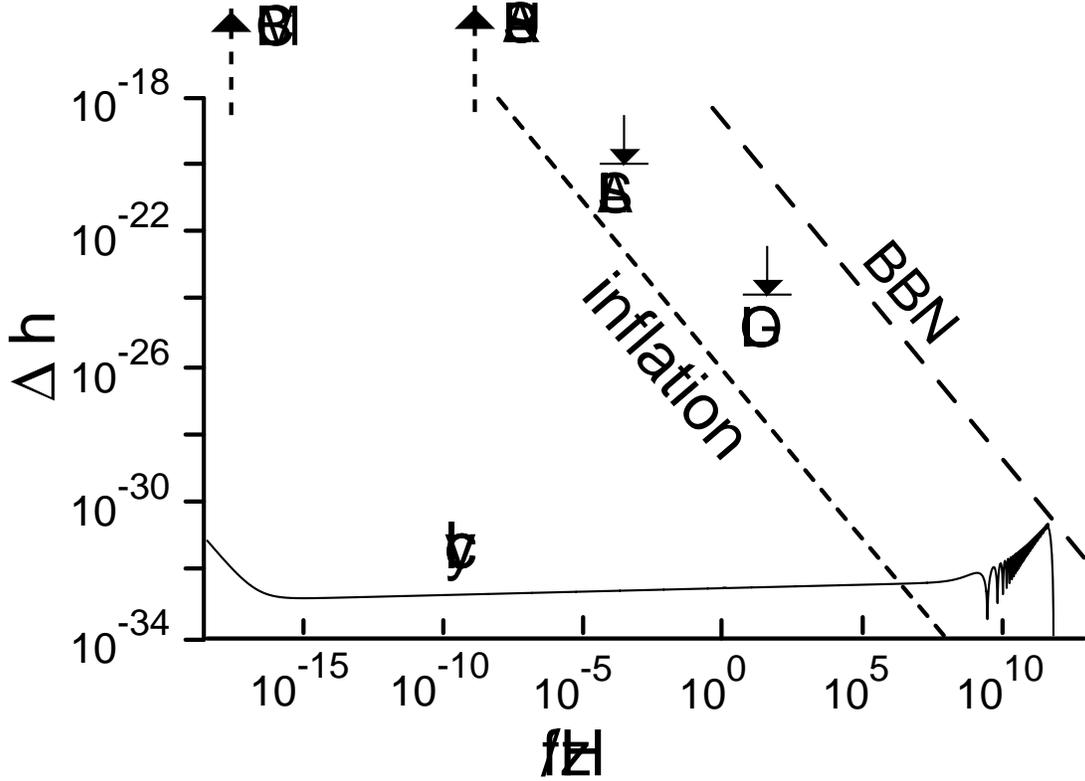


FIGURE 3. Plots of the dimensional strain Δh vs. frequency for the gravitational waves anticipated in cyclic (solid) and inflationary models (short-dashed) Included are constraints from big bang nucleosynthesis (BBN, excludes range to upper right of long-dashed line) and best limits anticipated from the proposed LISA and advanced LIGO gravitational wave detectors. The cosmic microwave background (CMB) and pulsar constraints are shown dashed to indicate that they lie beyond the top of the figure. Both models satisfy the BBN constraints for typical parameters. CMB is currently the most promising approach for observing inflationary gravitational waves. The cyclic prediction is orders of magnitude below current and projected tests.

CONSTRUCTING THE CYCLIC POTENTIAL

We have summarized the basic ingredients needed to compute the spectrum of density perturbations, and the reader can look to Refs. [14, 17] for the technical details. But how does this consideration constrain the effective potential for the cyclic model? For inflation, the most stringent constraints are on the flat part of the potential, the range of the inflaton field where the density perturbations are generated. The constraints are commonly expressed as bounds on two “slow-roll” parameters:

$$\varepsilon = \left(\frac{V'}{V}\right)^2 \ll 1 \text{ and} \quad (3)$$

$$\eta = \frac{V''}{V} \ll 1. \quad (4)$$

For the cyclic model, the analogous constraints are on the steep portion of the potential where perturbations are generated. See Fig. 2. The constraints can be expressed in terms of two “fast-roll” parameters [14, 15]:

$$\bar{\epsilon} = \left(\frac{V}{V'} \right)^2 \ll 1 \text{ and} \quad (5)$$

$$\bar{\eta} = 1 - \frac{V''V}{(V')^2} \ll 1. \quad (6)$$

The first constraint forces the slope to be steep and the second fixes the curvature, where each applies to the range of ϕ where the fluctuations are generated that are within the horizon today.

The result is that the constraints in the two models are remarkably similar. The bad news is that the degree of tuning in cyclic models is no better than in inflationary models. The good news is that it is no worse. Also, recall, the cyclic model avoids the need for an inflaton, the inflation potential, and two episodes of accelerated expansion. So, there is actually some net gain for cyclic models, even if not in terms of degree of tuning. A further discussion of designing potentials for cyclic models can be found in [15].

In sum, the cyclic model has rapidly developed into a promising and provocative alternative to the standard big bang inflationary picture. There remain open issues, most especially a rigorous demonstration that the bounce can occur when quantum fluctuations are included. (A related, unproven issue for inflationary cosmology is a rigorous demonstration that the universe emerges from the big bang with the right conditions to have inflation.) But, other than this uncertainty, the other key aspects of the model are well-developed in technical detail. So, it appears for the next few years, there will be continued development of the models vigorous debates about which is theoretically preferable. Ultimately, though the answer must be determined by observation, either the detection of primordial gravitational waves or of other distinguishing features yet to be determined. What is at stake is nothing less than our understanding of the past history and future fate of the universe.

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