Answer THREE questions

The numbers in square brackets in the right hand margin indicate the provisional allocation of maximum marks per sub–section of a question.

You may assume the following (using standard notation):

Constant of gravitation G$= 6.672 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}$
Radiation constant $\alpha = 7.565 \times 10^{-16}\,\mathrm{J\,m^{-3}\,K^{-4}}$
Speed of light c$= 3 \times 10^{8}\,\mathrm{m\,s^{-1}}$
1 Mpc $= 3.086 \times 10^{19}\,\mathrm{km}$
1 yr $= 3.156 \times 10^{7}\,\mathrm{s}$

1. By considering the first law of thermodynamics in the form

$$dE + p\,dV = T\,dS$$

applied to an expanding volume V of physical radius a, derive the fluid equation as given by

$$\dot{\rho} + 3\,\frac{\dot{a}}{a}\left(\rho + \frac{p}{c^2}\right) = 0$$

which describes the evolution of the density ρ where p is the pressure. [6]

Explain the physical significance of the two terms inside the brackets as the volume of the Universe increases. [3]

The Friedmann equation is given by

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2}$$

where a is the scale factor and k is the curvature term.

Assuming that the Universe is flat and radiation-dominated, solve the Friedmann and fluid equations to obtain:

$$a(t) = \left(\frac{t}{t_0}\right)^{1/2}; \qquad \rho(t) = \frac{\rho_0}{a^4} = \rho_0\left(\frac{t_0}{t}\right)^2$$

where you may assume that the pressure of radiation is given by $\rho c^2/3$. [7]

For a Universe containing both matter and radiation, discuss the evolution of the matter density ρ_m and radiation density ρ_r assuming in turn that each separately dominates. You may assume that for matter domination, $a \propto t^{2/3}$ and $\rho_m \propto 1/a^3$. [4]

2. The Friedmann equation can be written in the form

$$H^2(t) = \left(\frac{\dot{a}(t)}{a(t)}\right)^2 = \frac{8\pi G}{3}\rho(t) - \frac{k}{a^2(t)}$$

where $H(t)$, the Hubble parameter represents the expansion rate of the Universe, $a(t)$ is the scale factor, $\rho(t)$ is the matter density and k is the curvature term.

By inspection of this equation, discuss the different behaviours with time of the Universe for the following four models: [10]

(a) $k = 0,\ a \propto t^{2/3}$;

(b) $k = -1,\ \rho = 0$;

(c) $k = -1,\ \rho > 0$;

(d) $k = +1$.

Define what is meant by the *density parameter* Ω and show how the Friedmann equation can be re-written in terms of Ω. [5]

Estimate the age of the Universe for case (a) above if $H_0 = 70\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$. [3]

Briefly discuss this result in the light of evidence that there are some globular clusters with ages of 11.5 Gyr and outline how a non-zero cosmological constant can solve this problem. [2]

3. Explain why we now believe that the Universe is accelerating. Your account should include a description of the background and techniques used to make this measurement. [7]

Describe in simple terms the physical meaning of the cosmological constant Λ and why it produces an acceleration. [3]

Explain why we now know that the Universe has a flat geometry. Again, your account should include a description of the background and techniques used to make this measurement. [7]

Explain the two fundamental problems raised by the presence of a cosmological constant in our Universe. [3]

4. Using the Friedmann equation as given in Question 1, and by assuming that the Universe is radiation-dominated (i.e. $a \propto t^{1/2}$) with negligible curvature and that the density $\rho = \alpha T^4/c^2$, show that the temperature of the Universe T is related to age t via: [5]

$$T^2 = \frac{c}{2t}\left(\frac{3}{8\pi G\alpha}\right)^{1/2}$$

Determine the temperature and age of the Universe when its density equalled that of water ($= 1000\,\mathrm{kg\,m^{-3}}$). [3]

Two of the important epochs in the evolution of the Universe are the eras of nucleosynthesis and decoupling. Describe the conditions and the physical processes that occur at these two eras. [12]

5. Describe *three* independent pieces of observational evidence that support the Hot Big Bang model. [6]

Explain what is meant by the 'horizon and flatness problems' and why they represent failures of the Big Bang model. [8]

Describe how the concept of inflation can solve both of these problems. [6]