**Answer THREE questions**

The numbers in square brackets in the right hand margin indicate the provisional allocation of maximum marks per sub–section of a question.

You may assume the following (using standard notation):

Critical density $\rho_c = 3H_0^2/8\pi G$

Hubble parameter $H = \dot{a}/a$

Lambda parameter $\Omega_\Lambda = \Lambda/3H_0^2$

1 Mpc $= 3.086 \times 10^{19}$ km

1 yr $= 3.156 \times 10^7$ s

Gravitation constant $G = 6.670 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}$

Solar mass $M_\odot = 1.989 \times 10^{30}$ kg

1. By considering the potential and kinetic energies of a particle acting under Newtonian gravity, derive the Friedmann equation for an expanding Universe in the form:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2}$$

where $a$ is the scale factor, $\rho$ is the density and $k$ is a constant. **[10]**

By differentiating the full Friedmann equation, as given by

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} + \frac{\Lambda}{3},$$

and using the fluid equation, given by

$$\dot{\rho} + 3\frac{\dot{a}}{a}\left(\rho + \frac{p}{c^2}\right) = 0,$$

show that for a pressure-less universe, the deceleration parameter $q_0$ is given by

$$q_0 = \frac{\Omega_0}{2} - \Omega_\Lambda(t_0)$$

where **[10]**

$$q_0 = -\frac{a(t_0)\ddot{a}(t_0)}{\dot{a}^2(t_0)}.$$

2. Define what is meant by the *density parameter* $\Omega$ and calculate the critical density of the universe if $H_0 = 70\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$. **[3]**

   If the density of the universe is dominated by galaxies with an average mass of $10^{11}\,\mathrm{M_\odot}$ and separation of $2\,\mathrm{Mpc}$, calculate $\Omega$. **[3]**

   Explain how the presence of 'dark matter' may be required to account for (a) the rotation curves of spiral galaxies; and (b) the mass of clusters of galaxies. **[7]**

   Explain to what extent this dark matter can be baryonic, and what forms it could take. **[3]**

   Discuss the likely forms of non-baryonic dark matter. **[4]**

3. Describe in detail the formation of the Cosmic Background Radiation (CBR). **[8]**

   Describe how recent observations of the CBR with the BOOMERANG experiment have measured the geometry of the Universe. **[7]**

   At the epoch of decoupling, the scale factor $a$ of the Universe was one-thousandth of its present value. Assuming that the Universe is always matter-dominated (i.e. $a \propto t^{2/3}$), calculate the age of the Universe at decoupling. [You may assume $H_0 = 70\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$.] **[5]**

4. Describe how primordial nucleosynthesis occurs in the early Universe and indicate how $^4$He is formed. **[10]**

   By making the simplifying assumptions that the only elements produced are $^1$H and $^4$He, and that all the neutrons are in $^4$He, calculate the fraction of the total mass of the Universe in the form of $^4$He. You may assume that neutrons are converted into protons between ages of 1.4 and 400 s, and the neutron has a half-life of 400 s. **[5]**

   Explain why (a) the observed abundance of deuterium is so sensitive to the density of baryonic material in the universe; and (b) why it has been impossible to produce elements beyond $^7$Li in the Big Bang nucleosynthesis. **[5]**

5. The Friedmann equation can be written as an equation showing how the density parameter $\Omega$ varies with time

$$|\,\Omega(t) - 1\,| \;=\; \frac{|\,k\,|}{a^2\,H^2}.$$

Using the solutions for radiation- and matter-dominated Universes ($a \propto t^{1/2}$; $a \propto t^{2/3}$), show how $\Omega$ varies with time, and explain the ensuing "flatness problem". **[7]**

Assuming a radiation-dominated Universe, estimate how close $\Omega$ was to unity at an age of $t = 10\,\mathrm{s}$, if today ($t_0 = 1.4 \times 10^{10}\,\mathrm{yr}$) we measure $\Omega_0 = 0.3$. **[3]**

By considering the Friedmann equation as given above, explain how the concept of inflation can solve the flatness problem. **[4]**

Describe two other failures of the Big Bang model and explain how inflation can solve these problems. **[6]**