Answer THREE questions

The numbers in square brackets in the right hand margin indicate the provisional allocation of maximum marks per sub–section of a question.

You may assume the following (using standard notation):

$$1\ \mathrm{Mpc} = 3.086 \times 10^{19}\,\mathrm{km}$$
$$1\ \mathrm{yr} = 3.156 \times 10^{7}\,\mathrm{s}$$

1. By considering the potential and kinetic energies of a particle acting under Newtonian gravity, derive the Friedmann equation for an expanding Universe in the form: [10]

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2}$$

where a is the scale factor, ρ is the density and k is a constant.

By considering the velocity of recession and Hubble's law, further show that the Hubble parameter is given by: [3]

$$H = \frac{\dot{a}}{a}$$

The second important equation in cosmology is the fluid equation which describes the evolution of the density ρ and is given by

$$\dot{\rho} + 3\,\frac{\dot{a}}{a}\left(\rho + \frac{p}{c^2}\right) = 0.$$

where p is the pressure.

Assuming that the Universe is flat and matter-dominated, solve the Friedmann and fluid equations to obtain: [7]

$$a(t) = \left(\frac{t}{t_0}\right)^{2/3}; \quad \rho(t) = \rho_0\left(\frac{t_0}{t}\right)^2; \quad \text{and}\ \ H = \frac{2}{3t}.$$

2. One of the fundamental problems in cosmology is the accurate measurement of the Hubble constant. Describe how the extragalactic distance scale is determined using primary and secondary distance indicators, and explain briefly how observations with the *Hubble Space Telescope* have improved the calibration of the distance ladder. [17]

   Studies of Globular Clusters in our Galaxy set a lower limit to the age of the Universe of 11.5×10^9 yr. Using this age and the solution for a matter-dominated, flat Universe, as given in Question 1, determine an upper limit to the Hubble constant H_0, in units of $\mathrm{km\,s^{-1}\ Mpc^{-1}}$. [3]

3. Describe in detail the formation of the Cosmic Background Radiation. [10]

   The *COBE* satellite detected small temperature fluctuations in the Cosmic Microwave Background Radiation. Describe the simple picture of how structure in the Universe can grow from these fluctuations. [4]

   Explain why this simple picture does not work and how non-baryonic dark matter can solve the problem. [6]

4. Describe how primordial nucleosynthesis occurs in the early Universe and indicate how ^{4}He is formed. [10]

   By making the simplifying assumptions that the only elements produced are ^{1}H and ^{4}He, and that all the neutrons are in ^{4}He, calculate the fraction of the total mass of the Universe in the form of ^{4}He if the final proton-to-neutron ratio is 7. [4]

   Explain why the observed abundances of D, He and Li require the Universe to contain large amounts of non-baryonic dark matter if it possesses the critical density. [6]

5. Describe the *three* independent pieces of observational evidence that support the Hot Big Bang model. [6]

The Friedmann equation can be written as an equation showing how the density parameter Ω varies with time

$$|\,\Omega(t) - 1\,| \;=\; \frac{|\,k\,|}{a^2\,H^2}$$

where k is the curvature term, a the scale factor, and $H = \dot{a}/a$ the Hubble parameter.

Using the solutions for radiation- and matter-dominated Universes ($a \propto t^{1/2}$; $a \propto t^{2/3}$) and ignoring the curvature term, show how Ω varies with time, and explain the ensuing "flatness problem". [7]

Assuming for simplicity that the Universe is radiation-dominated, use the equation that you have derived above to estimate how close Ω was to unity at the epoch of nucleosynthesis (age, $t = 1\,\mathrm{s}$), if today ($t_0 = 10^{10}\,\mathrm{yr}$) we measure $\Omega_0 = 0.7$. [3]

By considering the Friedmann equation as given above, explain how the concept of inflation can solve the flatness problem. [4]