1 Chapter One

1.1 Introduction

Cosmology is the attempt to understand the present state of the universe as a whole from both the observational and theoretical perspectives and thereby shed light on the origin and ultimate fate of the universe. Why is the universe structured today in the way that it is, how did it develop into its current form and what will happen to it in the future? Since humanity is also part of the universe, these questions apply equally to ourselves: where do we come from and where are we going? In addressing these questions, cosmologists draw on many different branches of physics, most notably from thermodynamics, nuclear physics, quantum mechanics, relativity and elementary particle physics.

1.2 Structure of the Course

The plan of this course is shown in Fig. (1.1). We begin with a survey of the observable universe, focusing on Hubble's law, and discuss the key properties of the cosmic microwave background. Having established that the universe s expanding, we will cover the derivation of the equations that describe cosmic evolution and show how they can be solved in a number of different settings. We will then consider how different cosmological models can be distinguished in terms of a number of observable parameters. This will allow us to determine the age of the universe and describe possible outcomes for its future destiny. We will then look into some recent evidence that the expansion of the universe is presently accelerating and explain how this can be understood in terms of a cosmological constant.

We will then look into the history of the universe, focusing on the success of the big bang model in explaining the origin of the microwave background and the successful prediction of the light element abundances. We will discuss a number of shortcomings of the model and explain how these are resolved by the inflationary scenario. We will see how the physics of inflation is related to the idea of a cosmological constant. The evidence for dark matter in the universe will be discussed and, finally, we will look into the origin of structure (galaxies and clusters). If time permits, we will see how this is related to the temperature fluctuations in the cosmic microwave background.

1.3 Structure of the Universe

In cosmology, the unit for measuring distance in the universe is the megaparsec (Mpc), i.e., a million parsecs. One parsec is 3.26 light years and one light year (the distance light can travel in a year) is $9.463 \times 10^{15}\,\mathrm{m}$. Thus,

$$1\,\mathrm{Mpc} = 3.086 \times 10^{22}\,\mathrm{m}$$

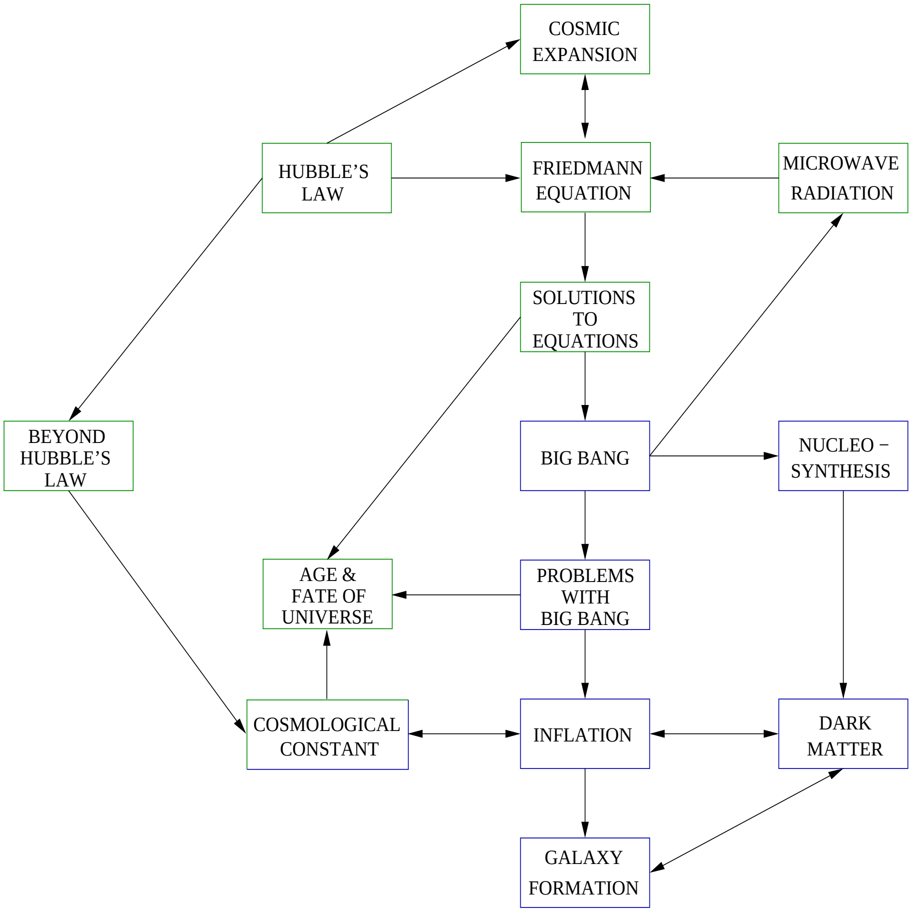

Fig. (1.1). The structure of the course.

Stars: Stars are the main source of visible light in the universe. Stellar masses vary in the range $0.01M_\odot$ to $100M_\odot$, where $1M_\odot = 2\times10^{30}$ kg is the mass of the sun (solar mass). The nearest stars to the sun are a few light years away.

Galaxies: Stars do not behave as separate entities but group together under the attractive force of gravity into galaxies. Our sun belongs to the Milky Way Galaxy. The Milky Way is comprised of a central dense region of stars known as the 'bulge' with a number of spiral arms attached to it. These arms also contain numerous stars. The radius of the bulge is about 10^4 ly and the arms extend outwards by at least 12.5 kly. We are located some 28,000 ly from the centre of the galaxy. Cosmology is interested primarily in the large scale structure of the universe. Over such large scales, the internal structure of galaxies is unimportant. In most of this course, we shall treat galaxies simply as point–like objects that have a mass and emit light but no internal structure. The mass of a typical galaxy in the universe is in the range $10^{10}M_\odot$ to $10^{11}M_\odot$. There are as many as 10^{11} galaxies in the observable universe.

Clusters of Galaxies: Although galaxies may be viewed as separate, gravitationally bound objects, they group together due to their mutual gravitational attraction

Object	Mass	Size
Spiral Galaxy	$10^{11} M_\odot$	10^{-2} Mpc
Distance between Neighbouring Galaxies	—	1 Mpc
Clusters of Galaxies	$10^{13} M_\odot$	1 Mpc
Superclusters	$10^{15} M_\odot$	10 Mpc
Voids	—	$10 - 100$ Mpc
Observable Universe	$10^{22} M_\odot$	10^4 Mpc

Table (1.3): Mass and length scales of objects in the universe.

into clusters. Our Milky Way is in what is called the Local Group – the largest galaxy in this group is the Andromeda galaxy, some 770 kpc away from our Galaxy. Some galaxy clusters contain only a few galaxies, whereas others may hold many thousands. Typically a galaxy cluster occupies a volume of a few Mpc^3. The typical separation between a galaxy is (roughly) 1 Mpc.

Superclusters and Voids: Galaxy clusters also group into what are known as superclusters of galaxies. The typical scale associated with a supercluster of galaxies is 100 Mpc. In between these superclusters are voids. Voids appear to be devoid of any matter and can be as large as $50 - 100$ Mpc.

Large Scale Smoothness: Over distance scales in excess of 100 Mpc, the universe appears to become smooth, in the sense that structures larger than superclusters of galaxies are not observed.

Observable Universe: The observable universe corresponds to the bit of the universe that we can ever hope to observe, even in principle. This distance is limited by the finite age of the universe and the speed of light. It is about $5\,\mathrm{Gpc} \approx 10^{26}\,\mathrm{m}$.

The key point is that the structure of the universe appears to be hierarchical:

$$\text{Stars} \longrightarrow \text{Galaxies} \longrightarrow \text{Clusters} \longrightarrow \text{Superclusters} \longrightarrow \text{Observable Universe}$$

Some typical scales are summarized in Table 1.3.

We now proceed to discuss two cornerstone observations that revolutionised our understanding of the structure and history of the universe. They are the observation that galaxies are receding from us and the discovery of the Cosmic Microwave Background.

1.4 Expansion of the Universe and Hubble's Law

Consider what happens when a wave is emitted by a source and is then picked up by a receiver positioned some distance away. If the distance between the emitter and receiver increases with time, each crest has to travel a little further than the one that went before it. This results in an *increase* in the distance between two neighbouring crests, i.e., the wavelength when measured at the receiver is slightly *longer* than that measured at the emitter. The overall effect of the relative separation of the

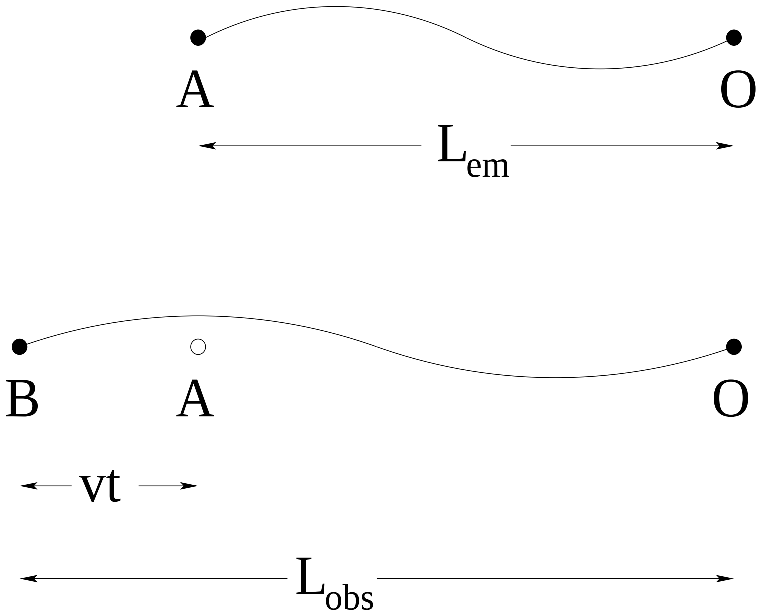

Fig. (1.2). The observed wavelength, $L_{\rm obs}$ is increased due to the motion of the source away from the receiver.

emitter and receiver is to increase the observed wavelength of the wave. This shift in the wavelength is known as the *Doppler effect* after the Austrian physicist who discovered it in 1842.

This effect can be quantified by considering a light source located at A in Fig. (1.2) and a receiver located at O. Suppose that initially the distance between the two is $\lambda_{\rm em}$ and that the source emits a single wavelength of light with this wavelength. The time taken for this wave to be emitted is $t = \lambda_{\rm em}/c$ (i.e. the inverse of its frequency). Suppose that the source is moving away from the (stationary) observer at O with a speed v. Then the distance it travels when emitting the wave is vt and, consequently, the wavelength of the wave when observed by O is actually[1] $\lambda_{\rm obs} = \lambda_{\rm em} + vt$, i.e.,

$$\frac{\lambda_{\rm obs}}{\lambda_{\rm em}} = 1 + \frac{v}{c} \tag{1.1}$$

The redshift, z, is then defined in terms of the ratio of the emitted and observed wavelengths:

$$z \equiv \frac{\lambda_{\rm obs}}{\lambda_{\rm em}} - 1 \qquad \Longrightarrow \qquad z = \frac{v}{c} \tag{1.2}$$

Since light has wavelike properties, its wavelength also increases if the emitter and detector are moving away from each other. We may think of light as being emitted by the galaxies at some point in the distant past and later received here on Earth by our telescopes. What would happen if all the galaxies were moving away from

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[1] This argument only works for speeds much less than the speed of light. There is a correction when $v \approx c$, but it is small for the effects we are considering in this Section.

us? We would expect the wavelength of the emitted light to be increased due to the Doppler effect.

This is what is indeed observed in the vast majority of galaxies. The redshift is measured by comparing the wavelengths of characteristic emission and absorption lines in the spectra. One of the most important is the wavelength corresponding to the $n = 2$ to $n = 1$ transition of the hydrogen atom. This is known as the Lyman–alpha (Lα) line. Its rest wavelength is 121.6 nm, corresponding to the far ultra–violet. The spectrum of a distant quasar is shown in Fig. 4 of the handout.

The discovery that the light from most galaxies is redshifted was made in 1914 by Slipher. This work was taken further by Edwin Hubble in the 1920's. As well as measuring the recession speed of distant galaxies from their observed redshift, Hubble was also able to establish their distance from us. He achieved this by measuring the brightness of a specific type of variable star known as a Cepheid variable. Such stars are pulsating and there is a strong correlation between their luminosity (brightness) and their period of oscillation. Hence, by measuring these parameters and their observed brightness, one can determine their distance from us.

By comparing the redshift and corresponding distance of the nearby galaxies, Hubble arrived at his law that *the recession velocity, v, of a galaxy is directly proportional to its distance, d, from us*:

$$v = H_0 d \tag{1.3}$$

where the constant of proportionality, H_0, is known as Hubble's constant. Note that this constant has dimensions of $[\mathrm{Time}]^{-1}$. Equivalently, at low speeds (relative to the speed of light), or equivalently small redshifts, $z \ll 1$, the distance is proportional to the redshift:

$$d = \frac{cz}{H_0} \tag{1.4}$$

The original data from Hubble's 1929 paper is shown in Fig. (1.3). Unfortunately Hubble underestimated the distances to the galaxies he was observing by some considerable margin and his empirical value of H_0 was too high by as much as a factor of ten. Indeed, such uncertainties plague modern estimates of the Hubble constant to such an extent that H_0 is still poorly determined. It is conventional to parametrise its value in terms of the parameter, h, defined such that

$$H_0 = 100h \,\mathrm{km\,s^{-1}\,Mpc^{-1}} \tag{1.5}$$

In terms of seconds or years, this becomes

$$H_0^{-1} = 3.1 \times 10^{17} h^{-1} \,\mathrm{sec} = 9.8 \times 10^{9} h^{-1} \,\mathrm{yr} \tag{1.6}$$

Current observations indicate a range of values $0.6 < h < 0.75$. For example, recent data from the Hubble Space Telescope indicates a best–fit value of

$$H_0 = 72 \pm 8 \,\mathrm{km\,s^{-1}\,Mpc^{-1}} \tag{1.7}$$

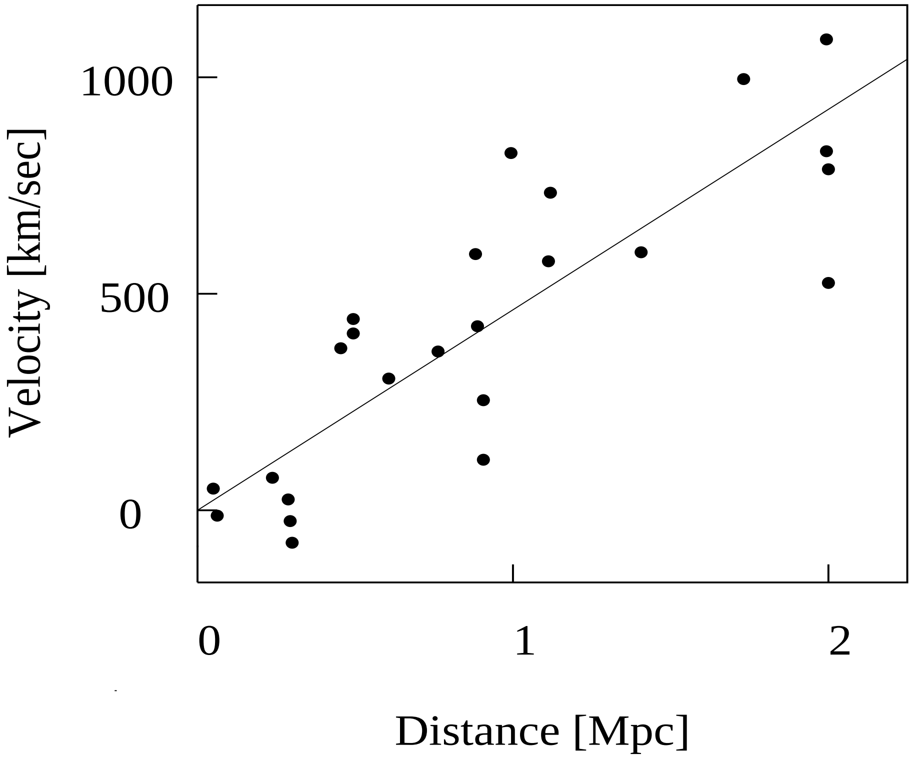

Fig. (1.3). The data from Hubble's original paper comparing the recession velocity of nearby galaxies with their redshift. Hubble deduced a value of $H_0 = 530\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ for the constant of proportionality. This is about a factor of ten too high, because he underestimated the distances involved.

- The key conclusion is the observed redshift of the vast majority of galaxies can be explained in terms of the motion of galaxies *away* from ours. This is a fundamental cosmological observation and it leads us to conclude that the universe is *expanding*.

1.5 Cosmic Microwave Background Radiation

The universe is bathed in electromagnetic radiation with wavelengths peaked in the microwave band of the spectrum. This radiation was discovered in 1965 and is known as the *cosmic microwave background*, or CMB for short. The CMB radiation reaches us from all directions in the universe and this indicates that its origin is cosmic, rather than due to some localized effect. If it arose from a process occurring in a specific region of the universe, we would expect its intensity to be sensitive to the direction in which we were observing, but this is not observed.

The observed spectrum of the CMB radiation (the dependence of its intensity on wavelength) is shown in Fig. 8 of the lecture handouts. The form of this spectrum matches that of a blackbody to a very high accuracy[2]. It is peaked and falls off exponentially at higher wavelengths. The location of the peak is uniquely determined by

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[2] A blackbody is a theoretical object that absorbs all energy incident up on it, reflects none

the temperature. This implies that the radiation has a characteristic temperature and the best–fit curve to the data corresponds to blackbody radiation with a temperature of $2.728 \pm 0.004\,\mathrm{K}$. Although the data is present in Fig. 8, you can not actually see it because it fits the best–fit curve so well and has been so accurately measured that the error bars are thinner than the thickness of the line! (Indeed, the error bars would have to be magnified over 200 times before they could be seen).

This is the most accurate blackbody to have ever been observed. As we shall see, the CMB is one of the corner stone predictions of the hot big bang model. It implies that when the universe was younger, it was *hotter* and effectively in a state of (near) thermal equilibrium.

Another remarkable feature of the CMB is that its temperature is uniform regardless of the direction from which it reaches us. Fig. 9 from the handout shows three full sky temperature maps of the CMB made by the Cosmic Background Explorer (COBE) satellite. COBE measured the temperature differences between two given directions separated by about 10^o. The maps are plotted in galactic coordinates, with the plane of the Milky Way running horizontally along the centre of the map. Thus, Sagittarius is in the center. In these plots, darker (lighter) regions denote lower (higher) temperatures.

The first map shows what would be observed if only temperature differences above a few mK could be detected – the temperature is uniform at this level of accuracy. It turns out that the radiation is warmer by 0.007K in one direction than in the opposite direction. This is due to a Doppler effect arising from the motion of our Milky Way relative to the rest of the universe. The inferred speed of our Galaxy is about $600\,\mathrm{km\,s^{-1}}$. Subtracting out this effect results in the third map – the radiation is *uniform to an accuracy of one part in 100,000.* (The broad red band along the centre of the plot is due to emission from the galactic plane and should be ignored).

To summarize, the CMB is very nearly, but not precisely, uniform to 1 part in 10^5.

1.6 Cosmological Principle, Isotropy and Homogeneity

Motivated by the above observations, we now come to a fundamental principle up on which modern cosmology is founded. This is known as the *Cosmological Principle* and states that *on sufficiently large scales the universe is both isotropic and homogeneous.* Isotropy implies that there is no preferred direction in the universe and, consequently,

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and emits energy with total efficiency. Blackbody radiation is the maximum energy that can be emitted by an object at a given temperature. In practice, blackbody conditions are relevant when the radiator is effectively isolated from its environment and in a state of (near) thermal equilibrium, as is the case for a star for example. This follows because if a body that is in thermal equilibrium absorbs radiation, all of this radiation must be subsequently re–emitted. If any radiation were to remain absorbed, the body would heat up and would then no longer be in thermal equilibrium. A hot object satisfying these conditions emits blackbody radiation in a range of wavelengths of differing intensities.

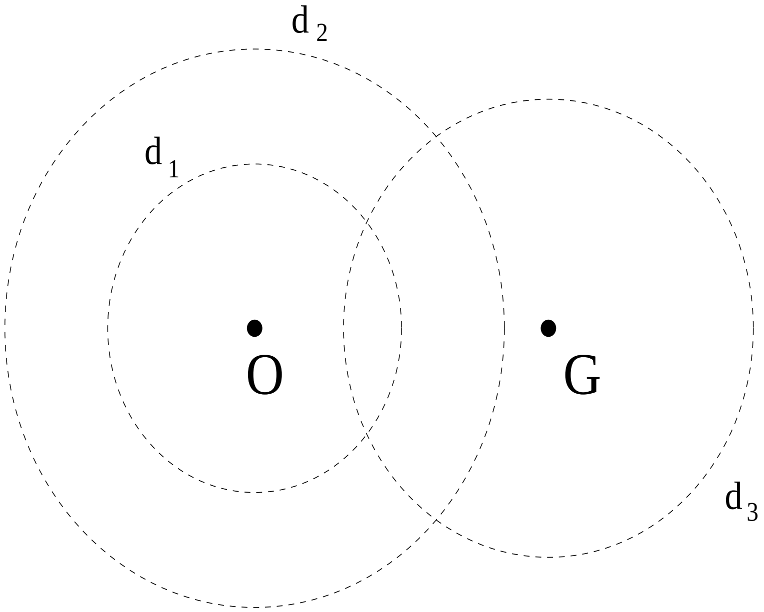

Fig. (1.4). If the universe is isotropic about two points in space, it is also homogeneous.

when the universe is viewed from a particular point, it looks the same regardless of direction. Homogeneity means that at a given instant the universe appears the same everywhere.

We must qualify by what we mean by 'sufficiently large scales'. Clearly, the cosmological principle does not hold on very small scales – a typical lecture theatre is neither isotropic nor homogeneous! Likewise, the internal structure of the Galaxy implies that the universe is neither isotropic nor homogeneous on scales below 10^4 pc. However, clusters of galaxies are evenly distributed throughout the universe and the universe does appear to become uniform on scales above 50 Mpc. Evidence for the isotropy of the universe follows from Hubble's law – the cosmic expansion and the recession of galaxies away from us is independent of the direction we look. Further evidence arises from the uniform temperature of the cosmic microwave background (Fig. 9).

We can not establish observationally that the universe is homogeneous, since we can only view the universe from one place (our Galaxy). However, the homogeneity of the universe follows directly from its isotropy if we assume that there is nothing special about our particular position in the universe[3]. Thus, if the universe appears isotropic to us, it should appear isotropic to a distant observer. To see why this implies that the universe is homogeneous, suppose that we are located at galaxy O in Fig. (1.4) and that a distant observer is located at galaxy G. Consider two shells of (hypothetical) matter that are centred around us at O. The isotropy of the universe implies that the

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[3] This is reasonable, since if there was something special about our location, then presumably we could not talk meaningfully about other regions of the universe and hence about cosmology in general.

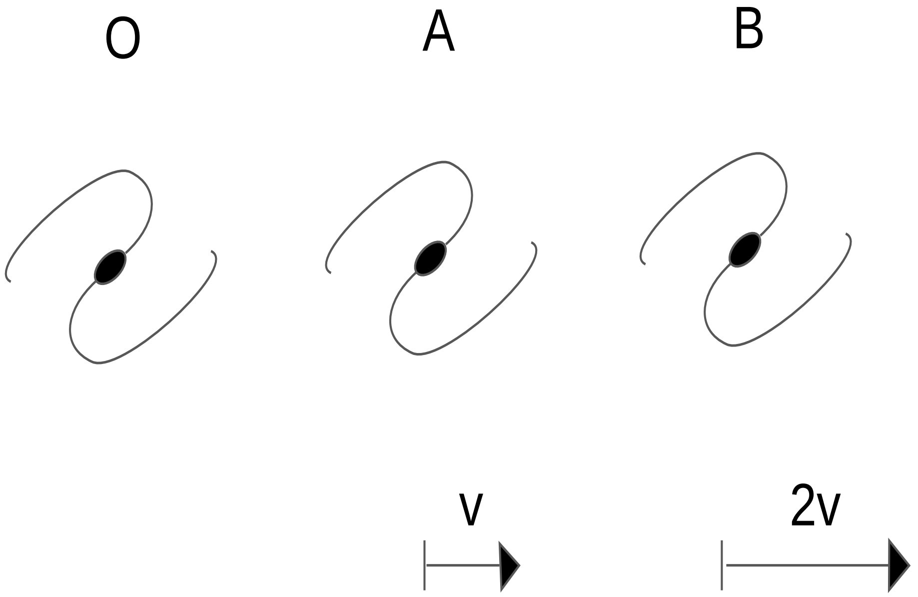

Fig. (1.5). When measured from galaxy O, the apparent velocity v of a galaxy grows in direct proportion to its separation distance. This follows as a direct consequence of the Cosmological Principle.

densities of these shells must be uniform. Let us denote these densities as d_1 and d_2, respectively. Now, by a similar argument, a shell of matter around G must also have a uniform density, d_3 say. If this shell intersects those around O, it follows that at the points of intersection $d_3 = d_1$ and $d_3 = d_2$. Hence, $d_1 = d_2 = d_3$ and, since the shells are arbitrary, the density of matter throughout the universe must be independent of position. This is what it means for the universe to be homogeneous.

It is important to emphasize that the Cosmological Principle is just a principle and that there is no *a priori* reason why it should apply to our universe. Nevertheless, by invoking the Cosmological Principle, we are able to make a number of simplifying assumptions when deriving and solving the cosmological equations that determine the evolution of the universe (see Chapter 2). In particular, physical quantities are independent of spatial position, so we can neglect spatial derivatives. Somewhat surprisingly, the solutions we find seem to describe the large–scale features of the universe in which we live.

1.7 Cosmological Principle and Hubble's Law

As we have discussed, from our vantage point in the Milky Way, it appears that the vast majority of other galaxies in the universe are moving away from ours, with a speed proportional to their distance from us given by Hubble's law (1.3). It is tempting to think of our Galaxy as positioned at some fixed point in space with the rest of the universe moving away from us, in much the same way that debris rushes away from the centre of an explosion. This is *not* correct because it places our Galaxy and ourselves at the very centre of the universe.

How, then, can we interpret Hubble's law? As it happens, Hubble's law follows as a direct consequence of the Cosmological Principle. If the universe is homogeneous (looks the same everywhere) then it follows that an observer in a distant galaxy should observe behaviour similar to that observed by us in the Milky Way, that is, he/she should observe all galaxies to be moving away from his/her own galaxy with a speed proportional to the separation distance.

In view of this, consider the idealized picture in Fig. (1.5) showing three galaxies, O, A and B, that for simplicity are evenly separated. Thus, galaxy B is twice as far from O as galaxy A. If A moves away from O at a speed v, it follows from the Cosmological Principle that an observer at A should observe galaxy B to be moving away from A at the same speed, v. Thus, galaxy B must be moving away from O at twice this speed, $2v$. Since B is twice as far from O as A is, this implies that the apparent separation speed of a galaxy from the observer increases in *direct proportion* to its distance – this is Hubble's law!

Finally, before concluding, we note for future reference that in view of the observed isotropy of the universe, a more precise way of writing Hubble's law is as a vector equation. As we look in a direction $\vec{r}$, the recession velocity of a galaxy a distance $|\vec{r}|$ from us is proportional to $\vec{r}$:

$$\vec{v} = H_0\vec{r} \tag{1.8}$$

Taking the modulus of both sides of this equation then results in Eq. (1.3)