10 Dark Matter: An Overview

This Section is a brief overview of the observed and inferred values for the Ω–parameter for baryonic and non–baryonic matter in the universe.

Visible Matter: The first estimate to make is the density of matter that is visible, i.e., the amount of material in stars. Our understanding of stellar structure is sufficient to deduce the mass of a star from its luminosity. By counting the number of stars in a sufficiently large and nearby region, we may estimate their density. The result is that the density of visible matter in the universe is much less than the critical density:

$$\Omega_{\rm visible} \approx 0.01 \tag{10.1}$$

Primordial Nucleosynthesis: We have seen in Section 9 that there are strong reasons from primordial nucleosynthesis for supposing that the density of baryons in the universe is constrained to be

$$0.011 \leq \Omega_B h^2 \leq 0.015 \tag{10.2}$$

where a subscript 'B' denotes the baryons. Since $h > 0.5$, then $\Omega_B < 0.06$ and there are not enough baryons in the universe to make up the critical density. Moreover, taking the favoured value of h to be 0.65 implies that $\Omega_B > 0.03$, implying that there is a lot more baryonic matter in the universe than that which is visible [cf. Eq. (10.1)]. In other words, most of the baryonic matter is dark.

Galaxy Rotation Curves: In general, galaxies rotate. A galactic rotation curve measures how the rotational velocity of the galactic disk varies with the distance from its centre. Since a galaxy is bound gravitationally, the centrifugal force arising due to the rotational velocity (angular momentum) of the matter in the outer regions of the galaxy must precisely balance the gravitational attraction towards the centre. Thus, the centrifugal force $F_C = mv^2/r$ must be balanced by the gravitational pull, $F_G = GMm/r^2$, i.e.,

$$v^2 \approx \frac{GM}{r} \tag{10.3}$$

where v represents the rotational velocity of the matter a distance r from the centre and M is the mass contained within this region. Note the mass outside this region does not contribute to the gravitational force due to Newton's 'iron–ball' theorem (see Section 2.1).

The rotational velocity is deduced by measuring the Doppler shift of light from different regions in a given galaxy. More specifically, it is measured by observing the Doppler shift of starlight and optical HII regions together with the cold HI clouds (21 cm). At sufficiently large distances from the centre, the velocity of interstellar gas is measured. On these larger scales, we would expect the mass contained within a distance r from the galactic centre to remain roughly constant, since most of the visible mass of a galaxy is concentrated towards its centre. In this case, Eq. (10.3) implies that the rotational velocity should fall as $v \propto r^{-1/2}$.

However, what is actually observed in numerous galaxies is that the rotational velocity stays *constant* at large distances. This implies that $M \propto r$ in the outer regions of the galaxy. A typical rotation curve, corresponding to the spiral galaxy NGC3198 is shown in Fig. (10.1). The rotation velocity remains roughly constant above an orbital radius of $8h^{-1}\,\mathrm{kpc}$ or so. The dashed line labeled 'disk' is what would be expected in the absence of any dark matter, i.e., the contribution expected from the luminous disk of the galaxy. The dashed line labeled 'halo' is the predicted contribution of a spherical halo of dark matter distributed around the galactic center. The two combined produce the solid curve that fits the data well. Fig. (10.2) shows the observed rotation curve of our own Milky Way Galaxy. The sun's orbital velocity, $v_\odot \approx 220\,\mathrm{km\,s^{-1}}$ at a radius of $r_\odot \approx 8\,\mathrm{kpc}$ is shown. This is about $60\,\mathrm{km\,s^{-1}}$ higher than it should be if the Galaxy contained only visible matter.

Consequently, it is concluded that galaxies are embedded in a spherical halo of dark matter. Typical observations indicate that the velocity is 3 times greater than expected [cf. Fig. (10.1) at large radii], implying there is ten times more matter than is seen directly. Thus, the density of galactic halos is estimated to be

$$\Omega_{\rm halo} \approx 10 \times \Omega_{\rm visible} \approx 0.1 \tag{10.4}$$

This is the second piece of evidence for dark matter. For the presently favoured value of the Hubble constant ($h = 0.65$), it is somewhat too high to be consistent with the upper limit on the baryonic Ω–parameter inferred from primordial nucleosynthesis, Eq. (10.2). This implies that some other form of 'non–baryonic' dark matter must be present.

Motions of Galaxy Clusters: Further evidence for non–baryonic dark matter follows from the dynamics of galaxy clusters. We can view galaxy clusters as gravitationally bound systems. Gravitational attraction causes the galaxies to move towards each other. These motions away from the motion due to the cosmic expansion are known as peculiar velocities. For example, the Milky Way is part of a cluster of galaxies dominated by the Andromeda galaxy (M31). Our galaxy is moving towards M31 at about $100\,\mathrm{km\,s^{-1}}$. The typical magnitudes of peculiar velocities observed in numerous clusters are in the range $300\,\mathrm{km\,s^{-1}} \leq \langle v_{\rm pec}\rangle \leq 1000\,\mathrm{km\,s^{-1}}$

By measuring peculiar velocities, one can infer the mass of the cluster. Since clusters are gravitationally bound, the Virial Theorem implies that the magnitude of the gravitational potential energy, $GM^2/2R$, of a cluster of mass M and size R must equal twice the kinetic energy of the cluster, i.e.,

$$\langle v_{\rm pec}\rangle^2 = \frac{GM}{2R} \tag{10.5}$$

Noting that the typical scale of a cluster is $R \approx 1 - 10\,\mathrm{Mpc}$, then Eq. (10.5) implies a mass scale of the order $10^{15} M_\odot$. Recalling that the number of galaxies in a typical cluster does not exceed around 10^3, this mass is about ten times higher than the expected combined contribution of galaxy masses (assuming a typical galactic mass is $\mathcal{O}(10^{11}M_\odot)$).

Overall, it is found that the density of matter associated with clusters of galaxies is about 30% that of the critical density, i.e.,

$$\Omega_{\rm cluster} \approx 0.3 \tag{10.6}$$

and this is comfortably in excess of the upper limit on the amount of baryonic matter (dark or otherwise) compatible with the primordial nucleosynthesis constraint. Thus, not only is most of the matter in the universe dark, it must be non–baryonic.

The Origin of Non–Baryonic Dark Matter: Broadly speaking, non–baryonic dark matter is matter that has mass but does not interact at today's energy scales directly with ordinary matter, either through the electromagnetic interaction or through the strong and weak nuclear forces. In particular, it does not interact with photons, so we can not 'see' it. Its only influence, therefore, is through gravity.

The currently favoured candidate for the non–baryonic dark matter in the universe is known as 'cold dark matter' (CDM). This is a hypothetical massive particle whose origin lies in Grand Unified theories of the fundamental interactions, such as superstring theory. Presently, the precise nature of the particle is unknown – however, for our cosmological purposes, this is not important. What is important is that the dark matter is said to be 'cold'. What this means is that the matter was behaving non–relativistically when it decoupled from (that is, fell out of equilibrium with) the rest of the matter in the very early universe sometime before the era of decoupling[23]. See Section 7.3 for a precise criteria for this to occur.

Now, we have seen previously from Eq. (7.12) that for a given type of particle, i, in equilibrium at a temperature, T, the distribution of momenta, p, is determined by

$$n_i(p)dp = \frac{4\pi g_i}{h^3}\frac{p^2 dp}{e^{E/k_BT} \pm 1} \tag{10.7}$$

where n_i is the number density of the particles with momenta p and E is the total energy of the particle, as given by Eq. (7.2). The '+' sign corresponds to a fermion particle and the '$-$' sign to a boson particle. The quantity g_i is a numerical constant and is called the 'degeneracy factor'.

Moreover, if the particle is non–relativistic, its mass–energy dominates its dynamics. Thus, we may approximate Eq. (7.2) in terms of Eq. (7.3):

$$E_{\rm total} = mc^2 + \frac{p^2}{2m} \tag{10.8}$$

Moreover, since $k_BT \ll mc^2$, we can neglect the ± 1 in the denominator of Eq. (10.7). Hence, the number density of these cold dark matter particles when they decouple at

[23]Conversely, the dark matter would be referred to as 'hot' if it fell out of equilibrium when it was still relativistic. However, hot dark matter is presently not favoured.

some temperature T is given by

$$n_{\rm cdm} = \frac{4\pi g_i}{h^3} \left(2mk_BT\right)^{3/2} e^{-mc^2/(k_BT)} \int_0^\infty dy\, y^2 e^{-y^2} \tag{10.9}$$

where we have defined a new variable

$$y^2 \equiv \frac{p^2}{2mk_BT} \tag{10.10}$$

The integral in Eq. (10.9) can be evaluated and has the numerical value $\sqrt{\pi}/4$. Hence, at the time when the cold dark matter particles decouple from the rest of the universe, their number density is given by

$$n_{\rm cdm} = \frac{g_i}{h^3} \left(2\pi mk_BT\right)^{3/2} \exp\left[-\frac{mc^2}{k_BT}\right] \tag{10.11}$$

Thereafter, the number density falls as $1/a^3$ as expected and for appropriate values of mass, m, and decoupling temperature, T, it is possible for these particles to contribute a density $\Omega_{\rm cdm} \approx 0.3$ at the present era. Since it is probable that these cold dark matter particles interacted with other matter in the early universe only through the weak nuclear force, they are generally referred to as 'Weakly Interacting Massive Particles', or WIMPs for short.

Inflation and Supernovae: We saw in Section 11 that inflation in the very early universe predicts that the present value of the total density in the universe should be very close to the critical density, i.e., Ω_0 should be very close to unity. This is a generic prediction of most of the realistic versions of inflation that have been proposed. Since the value of Ω inferred from galactic halos is considerably less than unity, this implies that most (but not all) of the dark matter in the universe is not confined to the galaxies, or indeed clusters of galaxies, and must be more evenly distributed throughout the universe. One of the simplest ways to achieve a smooth component of (non–baryonic) dark matter is to introduce a cosmological constant, Λ, such that $\Omega_{\rm matter} + \Omega_\Lambda = 1$. As we saw in Section 6, recent high redshift observations of type Ia supernovae indeed indicate that the universe is dominated today by a cosmological constant, where $\Omega_{\Lambda,0} \approx 0.7$.

Formation of Galaxies and Clusters of Galaxies: Here we just highlight an important piece of further evidence for non–baryonic dark matter that arises from the process that led to galaxy formation in the universe. We defer a more detailed discussion of this until Section 12, however, as the topic is a subject in its own right.

Summary: In a universe that underwent inflation, so that $\Omega_0 = 1$ today, only 1% of the matter in the universe is in the form of visible matter. No more than 10% is in the form of baryonic matter and the actual figure is probably somewhat less than this. Around 30% is in the form of non–baryonic matter that clumps together under the influence of gravity around galaxies and in clusters of galaxies. By far the dominant contribution is a cosmological constant, constituting 70% of the present density of the universe.