7 The Early Universe I

7.1 Overview

As we go back in time the universe becomes smaller and hotter. Moreover, the density of radiation scales as $\rho_{\rm rad} \propto a^{-4}$, whereas the matter scales as $\rho_{\rm mat} \propto a^{-3}$ and, of course, the cosmological constant remains fixed with time. Consequently, the radiation becomes progressively more important as we go back in time, since its density grows more rapidly than that of the matter. At sufficiently early times, it dominates over the matter and cosmological constant. The dependence on time of the scale factor at very early times is therefore expected to be given by Eq. (3.29), $a \propto t^{1/2}$. The scale factor vanishes at some finite time in the past and this is identified as the 'big bang'. Moreover, since $\rho_{\rm rad} \propto T^4 \propto t^{-2}$ from Eq. (3.15), it follows that the temperature of the radiation varies as the inverse of the scale factor:

$$T \propto \frac{1}{a} \qquad \Longrightarrow \qquad aT = \text{constant} \tag{7.1}$$

This is a very important relation and implies that $T \to \infty$ as $a \to 0$. We therefore infer that our universe began as a *hot big bang.*

The temperature at very early times would have been so high that stable matter as we know it today would not have existed. Atoms would have been ionized and indeed the temperatures were so high that even nuclei would have been unstable. The electrons, protons and neutrons that make up nuclei and atoms today would have behaved as free particles. The best way of viewing the early universe is therefore as a hot 'primordial soup' of radiation and elementary particles.

When discussing the physics of the early universe, it is often convenient to measure energy in terms of the particle physics unit of the electron volt (eV), where $1\,\text{eV} = 1.602 \times 10^{-19}\,\text{J}$. The mass of a given particle at rest can also be expressed in terms of this unit via Einstein's relation $E = mc^2$. In this case, it is referred to as the *mass–energy.* Furthermore, a given energy scale E can be converted to an effective temperature via the relation $E = k_B T$, where $k_B = 1.381 \times 10^{-23}\,\text{J}\,\text{K}^{-1} = 8.619 \times 10^{-5}\,\text{eV}\,\text{K}^{-1}$ is Boltzmann's constant.

7.2 Relativistic and Non–Relativistic Particles

A particle has two primary contributions to its energy. These are the energy arising from its mass and a contribution arising from its motion (momentum). In the very early universe particles may move at speeds close to the speed of light, so we must worry about relativistic effects. It turns out that these two contributions combine quadratically – the total energy of the particle, $E_{\rm total}$, is given by[17]

$$E^2_{\rm total} = m^2c^4 + p^2c^2 \tag{7.2}$$

[17] We are quoting the result here, because the derivation requires a full relativistic treatment.

Relativistic	Dominated by Kinetic Energy	$p > mc$ ($v \approx c$)	$k_B T > mc^2$
Non–relativistic	Dominated by Mass–energy	$p < mc$ ($v \ll c$)	$k_B T < mc^2$

Table (7.2): The conditions for a particle to be relativistic or non–relativistic can be expressed in terms of the temperature of the universe at a given time.

where m and p denote the mass and momentum of the particle, respectively. In the case where the particle is moving at speeds much less than the speed of light, the mass–energy dominates and we may write

$$E_{\text{total}} = mc^2 \left(1 + \frac{p^2}{m^2c^2}\right)^{1/2} \approx mc^2 + \frac{1}{2}\frac{p^2}{m} \tag{7.3}$$

where we Taylor expand the square root under the assumption that $p = mv \ll mc$. We see the first term is Einstein's equation $E = mc^2$ for the energy of a massive particle at rest and the second is just the expression for the kinetic energy of a particle.

We say that a particle is *relativistic* if its momentum dominates its mass–energy, i.e., $v \approx c$. It is *non–relativistic* if the mass–energy dominates the momentum, $v \ll c$. Another way to measure whether a particle is relativistic at any given time is to compare its mass–energy to the temperature of the universe. The temperature is a measure of the kinetic energy of the particles and the two are related by $E = k_B T$. Thus, a particle with mass m is relativistic if $k_B T \gg mc^2$ and is non–relativistic if $k_B T \ll mc^2$. (See Table 7.2). Thus, all massless particles are relativistic. Today, the only massless particles in the universe are the photon and three types of neutrino particles.

The distinction between relativistic and non–relativistic particles is important because it affects how the density of the particles varies. Since their mass–energy is negligible, relativistic particles behave like radiation and so their mass/energy density scales as (see Eq. (3.30))

$$\rho_{\text{rel}} \propto \frac{1}{a^4} \tag{7.4}$$

Non–relativistic particles, on the other hand, are dominated by their mass. Since they move slowly (compared to the speed of light), collisions between non–relativistic particles are rare and consequently, the pressure exerted by them is negligible. They therefore behave as a pressureless matter component with density scaling as the inverse of the volume of the universe:

$$\rho_{\text{nonrel}} \propto \frac{1}{a^3} \tag{7.5}$$

In other words, their density falls inversely with the volume since their (mass–)energy is constant.

7.3 Interactions between Particles

Elementary particles undergo various types of interactions in the early universe. For example, two particles may collide or a particle species may decay into lighter or massless particles. For a given interaction, the *cross–section*, σ, represents the probability of an interaction occurring over a given area. It has dimensions [length]^2. In the early universe the interactions occur between relativistic particles, so the volume available per unit time for an interaction to occur is given by

$$\sigma c \tag{7.6}$$

If the number density of the particle species under consideration is n, then the number of interactions occurring per unit time is

$$\sigma c n \tag{7.7}$$

and this implies that an interaction occurs once every

$$\tau = \frac{1}{\sigma c n} \tag{7.8}$$

Since the number density of particles scales as $n \propto a^{-3}$, the probability of particle collisions occurring becomes smaller (if the cross–section is constant). Interactions are said to occur if the time between interactions – i.e., the reaction rate – is *less* than the age of the universe at that time. We have seen in Section 5 that the Hubble time $H^{-1}(t)$ represents an upper limit to the age of the universe. The criteria for a given type of interaction to occur, therefore, is that

$$\tau = \frac{1}{\sigma c n} < H^{-1} \quad \Longrightarrow \quad \text{Interaction occurs} \tag{7.9}$$

$$\tau = \frac{1}{\sigma c n} > H^{-1} \quad \Longrightarrow \quad \text{Interaction has ceased} \tag{7.10}$$

The second statement follows since the inequality implies that you would have to wait longer than the age of the universe at that time for an interaction to occur.

7.4 Particles in the (Early) Universe

We now briefly summarize some of the different types of elementary particles that were present in the early universe.

Baryons: Baryons are not fundamental particles, but are comprised of three quarks. Protons, denoted p^+, and neutrons, denoted n^0, are examples of baryons. These particles consist of combinations of the so–called 'up' and 'down' quarks. There are many different types of baryons that are more massive than the neutron and proton, but these are very unstable and typically can only survive for about 10^{-12} s. Thus, only the neutron and proton can contribute significantly to the universe at late times and it is these two species that we have in mind when we talk about baryons. The mass–energy of the neutron is $m_n c^2 = 939.6\,\text{MeV}$ and this is slightly higher than the mass–energy of the proton, $m_p c^2 = 938.3\,\text{MeV}$. This mass difference plays an important role in the process of primordial nucleosynthesis (see Section 9).

Electrons: The electron is a fundamental particle (that is, it has no internal constituents). It is denoted e^- and has a mass–energy $m_e c^2 = 0.51\,\text{MeV}$. This is much less than that of the baryons. An important feature of the expansion of the universe is that it is governed by the force of gravity and not by the much stronger force of electromagnetism. Since the latter only operates between electrically charged bodies, this implies that the universe is electrically neutral. Hence, there must be one electron in the universe for every proton. Since the mass–energy of the electron is considerably less than that of the baryons, the electrons make a negligible contribution to the density of the universe today.

Radiation/Photons: When we talk about the extreme densities and temperatures of the early universe, it is often more convenient to think of electromagnetic radiation in terms of its quantum mechanical description of individual particles. These are called *photons* and are denoted by γ. A photon may be viewed as the particle equivalent of electromagnetic radiation. Since the energy of an electromagnetic wave increases as its wavelength decreases, a very high energy wave such as a gamma-ray has a very short wavelength. In this case, it is often more convenient to view the wave as a particle positioned where the crest of the wave would be. Photons have zero mass and electric charge and travel at the speed of light. The energy of a photon is related to its frequency, f, by

$$E = hf = pc \tag{7.11}$$

where h is Planck's constant and p is the corresponding momentum of the photon. Photons interact with all electrically charged matter such as baryons and electrons.

Neutrinos: Neutrinos are particles that are produced in some types of radioactive decay. There are three types of neutrino and they all interact extremely weakly with other particles. They are denoted by the symbol ν. In the standard model of particle interactions, they are massless and therefore travel at the speed of light, although there is now recent experimental evidence indicating they may have a mass. The standard assumption is that the neutrinos are massless and unless otherwise stated we assume this throughout the course.

Fermions and Bosons: One of the key properties of an elementary particle is its angular momentum, or spin. We may view spinning particles as rotating about an axis. The amount of spin that a particle carries determines its rate of rotation. The electron is an example of a spinning particle. The spin of all elementary particles is severely restricted. Those particles that do not rotate are said to have zero spin and this is one possibility. Rotating particles have a spin that is directly related to that of the electron.

The elementary particles are divided into two main groups depending on the amount of spin that they carry. These groups are referred to as *bosons* and *fermions*. These names honour the famous theoretical physicists Satyendra Bose and Enrico Fermi. Particles that have zero spin or twice the spin of the electron are examples of bosons. Particles that carry the same spin as the electron, or three times that amount, are called fermions. The electron is therefore an example of a fermion. Indeed, all neutrinos and baryons are also fermions. The photon, on the other hand, has twice the spin of the electron and is therefore a boson particle. The spin of the particle is important because it affects its energy density, as we discuss in the following subsection.

Antiparticles: Every particle has an antiparticle associated with it. A particle and antiparticle have the same mass and spin but *opposite electric charge*. (If the particle is electrically neutral, the antiparticle is as well). The antiparticle of the electron is called the positron and is denoted by e^+. Particles and antiparticles do not survive long in each others company. When a particle and antiparticle collide they annihilate each other and decay into two photons (representing packets of pure energy):

$$P + \bar{P} \quad \rightarrow \quad \gamma + \gamma$$

where $\bar{P}$ denotes the antiparticle of P. For example, the electron and positron annihilate:

$$e^- + e^+ \quad \rightarrow \quad \gamma + \gamma$$

What is important for our study of the early universe is that the reverse reaction is also possible at sufficiently high temperatures:

$$\gamma + \gamma \quad \rightarrow \quad P + \bar{P}, \qquad \text{for} \qquad k_B T > mc^2$$

The energy of a photon at temperature T is typically of the order of $k_B T$. Thus, if the energy of the photon exceeds the mass–energy of the particle, mc^2, it is energetically possible for the pure energy (photon) to be converted into mass. This is an important process in early universe physics – it establishes an equilibrium between the matter and radiation. If too much radiation is present, some of it will decay into particles and vice–versa. In this way, the distribution of particles in the early universe reaches thermal equilibrium.

7.5 Particles in Thermal Equilibrium

At the very earliest times in the history of the universe, the reaction rates between the different species of particles would have been sufficiently high that the particles would have been in mutual equilibrium. A given particle species would not fall out of equilibrium until it stopped interacting with the other particles and/or radiation.

The key point is that if thermal equilibrium holds the abundance and distribution of a particle species can be uniquely determined in terms of the known formula from quantum statistics. Here we have to quote the result. For a given type of particle, i, in equilibrium at a temperature, T, the distribution of momenta, p, is given by

$$n_i(p)dp = \frac{4\pi g_i}{h^3}\frac{p^2 dp}{e^{E/k_BT} \pm 1} \tag{7.12}$$

where n_i is the number density of the particles with momenta p and E is the total energy of the particle, as given by Eq. (7.2). The '+' sign corresponds to a fermion particle and the '−' sign to a boson particle. The quantity g_i is a numerical constant and is called the 'degeneracy factor'. Its numerical value is determined by the type of particle. For neutrinos, $g_i = 1$ and for all other particles we consider (protons, electrons, photons, etc.), $g_i = 2$.

The total energy density of the distribution is therefore given by multiplying Eq. (7.12) by the energy $E_i(p)$ of the particles and integrating over all possible momenta:

$$\epsilon_i = \int_0^\infty E_i(p) n_i(p) dp \tag{7.13}$$

In the relativistic limit, where the energy of the particle with momentum p is $E \approx pc$, the total energy density of the species is given by

$$\epsilon_i = \frac{4\pi g_i c}{h^3}\int_0^\infty \frac{p^3 dp}{e^{pc/k_BT} \pm 1} \tag{7.14}$$

We have actually met this integral before in Section 3.2, where we considered the energy density of the microwave background (see also Appendix B).

The total energy density of all particle species in equilibrium is then given by summing over the different types of particles present in the universe at any given time:

$$\epsilon = \sum_i \epsilon_i \tag{7.15}$$

Now, there is an important trick that allows us to calculate the energy density of a fermion–type particle in the relativistic limit. The trick is to note that

$$\frac{1}{e^x + 1} = \frac{1}{e^x - 1} - \frac{2}{e^{2x} - 1} \tag{7.16}$$

as is most readily verified by starting from the right–hand side and showing that it reduces to the left–hand side. We may therefore express the energy density of a species of fermionic particles as

$$\begin{aligned} \epsilon_i &= \frac{4\pi g_i c}{h^3}\int_0^\infty \frac{p^3 dp}{e^{pc/k_BT} + 1} \\ &= \frac{4\pi g_i c}{h^3}\left[\int_0^\infty \frac{p^3 dp}{e^{pc/k_BT} - 1} - \int_0^\infty \frac{2p^3 dp}{e^{2pc/k_BT} - 1}\right] \end{aligned} \tag{7.17}$$

In the second term on the right–hand side of Eq. (7.17), we may make the substitution $q = 2p$, since the variable p is a dummy variable. Hence, Eq. (7.17) becomes

$$\begin{aligned} \epsilon_i = \frac{4\pi g_i c}{h^3}\left[\int_0^\infty \frac{p^3 dp}{e^{pc/k_BT} - 1} - \frac{1}{8}\int_0^\infty \frac{q^3 dq}{e^{qc/k_BT} - 1}\right] \\ = \frac{4\pi c}{h^3}\left(\frac{7}{8}g_i\right)\int_0^\infty \frac{p^3 dp}{e^{pc/k_BT} - 1} \end{aligned} \tag{7.18}$$

The integral in Eq. (7.18) now corresponds to that for a relativistic bosonic–type particle at a temperature T (due to the presence of the -1 in the denominator). The only difference is that the degeneracy factor has acquired a corrective factor of 7/8. We may therefore conclude that *a relativistic fermionic particle with a degeneracy factor g_i that is in equilibrium at a temperature T behaves just as a bosonic particle at the same temperature, but with an effective degeneracy factor of* $(7/8)g_i$. We have already seen how the integral in Eq. (7.18) can be evaluated when we considered the energy density of the microwave background in Section 3.2 (see Appendix B for the details). There we found that the energy density of radiation (photons) at a temperature T is given by $\epsilon_{\rm rad} = \alpha T^4$, where α is the radiation constant (3.16). Performing a similar analysis to that of Section 3.2, we may introduce the variable $x \equiv pc/k_BT$. This implies that Eq. (7.18) reduces to

$$\epsilon_i = \frac{4\pi k_B^4 T^4}{c^3 h^3}\left(\frac{7}{8}g_i\right)\int_0^\infty \frac{x^3 dx}{e^x - 1} \tag{7.19}$$

The integral in this expression is evaluated in Appendix B, where it is shown to have the numerical value of $\pi^4/15$. Hence, the energy density of a relativistic fermionic particle species is

$$\epsilon_i = \frac{7 g_i}{16}\alpha T^4 \tag{7.20}$$

The crucial point is that for *all relativistic particles, the energy density scales as* $\epsilon_i \propto T^4$. Consequently, if there is a mixture of bosonic and fermionic particles, the total energy density of the 'soup' also depends on the fourth power of the temperature. It can therefore be expressed as

$$\epsilon_{\rm tot} = \frac{g_*}{2}\alpha T^4 \tag{7.21}$$

where

$$g_* \equiv \sum_{\rm boson} g_i + \frac{7}{8}\sum_{\rm fermion} g_i \tag{7.22}$$

is a constant or proportionality and is evaluated by summing over all the different boson and fermion particle species that are present at any given time. It is the total energy density, Eq. (7.21), that goes into the Friedmann equation:

$$H^2 = \frac{8\pi G}{3}\frac{\epsilon_{\rm tot}}{c^2} \tag{7.23}$$