3C74: TOPICS IN MODERN COSMOLOGY

Problem Sheet 1: Answers to be handed in by 10 February 2005

Question 1

The Friedmann equation describing the expansion of the Universe is given by

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2}$$

and the fluid equation, describing the evolution of the density ρ of material in the Universe is given by

$$\dot{\rho} + 3\frac{\dot{a}}{a}\left(\rho + \frac{p}{c^2}\right) = 0.$$

Using these two equations, derive the acceleration equation which describes the acceleration of the scale factor a and is given by

$$\frac{\ddot{a}}{a} = \frac{-4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right)$$

Question 2

Using the Friedmann equation as given above, determine the present density ρ_0 (in SI units) of the Universe asuming that it is flat and the Hubble constant $H_0 = 70\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$.

Estimate the present observed density of the Universe (in SI units) by supposing that the Milky Way is a typical galaxy containing 10^{11} stars (each weighing one solar mass) and that galaxies are typically separated by a distance of one megaparsec.

Compare the two densities you have derived and briefly comment on what this indicates about the type of Universe we live in.

$[1\,\mathrm{M}_\odot = 1.989 \times 10^{30}\,\mathrm{kg};\ 1\,\mathrm{pc} = 3.086 \times 10^{16}\,\mathrm{m};\ \mathrm{G} = 6.672 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}]$