COSMOLOGY ASTM108

PROBLEM SET 1

1 From the information provided in lectures, estimate the average density of matter in the universe and the corresponding average number density of protons in the universe.

2 A galaxy has a redshift of $z = 0.1$. How far away is it?

3 The relationship

$$z = \frac{v}{c} \tag{1}$$

between the redshift, z, of a galaxy and its recession speed, v, as derived in the lectures, is only valid at low speeds, $v \ll c$, or equivalently $z \ll 1$. At high velocities, special relativistic effects must be accounted for. The special relativity result is

$$1 + z = \sqrt{\frac{1 + v/c}{1 - v/c}} \tag{2}$$

Verify Eq. (2) reduces to Eq. (1) at sufficiently low speeds.
The Lyman–α line of quasar PC $1247 + 3406$ (discussed in lectures) is redshifted to 717 nm. How far away is it? What does this tell us about the age of the universe? (Actually for distant objects in cosmology, further considerations need to be made – in particular concerning light propagation time, but your answer should be a good indication of the scales involved).

4 The expansion of the universe is not precisely uniform – the Cosmological Principle holds only on sufficiently large scales. As well as their motion due to the cosmic expansion, galaxies exhibit what is termed 'peculiar velocity' arising from the gravitational attraction of their neighbours. For example, our galaxy is moving toward the Virgo cluster at a speed of $600\,\mathrm{km\,s^{-1}}$. Assuming this to be a typical peculiar velocity, estimate how far away a galaxy needs to be before it can be employed to determine Hubble's constant to within a 10% accuracy. (Assume the distance and redshift of the galaxy are known precisely).

5 The equation of state is

$$p = (\gamma - 1)\rho c^2 \tag{3}$$

where p and ρ represent the pressure and mass density of matter in the universe.

Why are ranges of values $\gamma < 0$ and $\gamma > 2$ considered to be unphysical?
Assuming $0 \leq \gamma \leq 2$, solve the conservation equation to deduce the variation of density with the scale factor. Assuming $k = 0$, solve the Friedmann equation to deduce how the scale factor and density both vary with time. For what values of γ is the expansion of the universe accelerating (i.e. $d^2a/dt^2 > 0$)? Deduce the critical value of γ such that the expansion rate is neither accelerating nor decelerating (i.e. $d^2a/dt^2 = 0$), and find the variation of the scale factor with time for this value of γ for an arbitrary value of k. This cosmological model is called the 'coasting universe'.

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