COSMOLOGY ASTM108
PROBLEM SET 4

1 (a) Show that for a flat universe dominated by pressureless matter, the dependence of the Hubble parameter on redshift is given by

$$H(z) = H_0(1+z)^{3/2} \tag{1}$$

(b) Hence show that the proper distance of a galaxy at redshift z at the present time is

$$D_P = \frac{2c}{H_0}\left(1 - \frac{1}{\sqrt{1+z}}\right) \tag{2}$$

(c). By employing your answer to part (b), verify that at the time of emission, the proper distance of a galaxy at redshift z was changing at the rate

$$\frac{dD_P}{dt} = 2c\left[\sqrt{1+z} - 1\right] \tag{3}$$

(d). By Taylor expanding your answer to part (b) around small redshifts, find an expression for the proper distance to quadratic order in redshift and deduce the value of the deceleration parameter, q_0, for this cosmological model.

2. (a) Consider a universe dominated by matter with an equation of state

$$p = (\gamma - 1)\rho c^2 \tag{4}$$

for some constant γ. From the definitions of the deceleration parameter and Ω–parameter, show that these two parameters are related by

$$q = \frac{1}{2}(3\gamma - 2)\Omega \tag{5}$$

regardless of the curvature of the universe. [Hint: whenever the deceleration parameter appears in a question, its probable that you will need the acceleration equation at some point].

(b). Consider a spatially flat universe containing pressureless matter and a cosmological constant. Verify that the deceleration parameter is given by

$$q = \frac{\Omega_M}{2} - \Omega_\Lambda \tag{6}$$

where the Ω_M and Ω_Λ are the Ω–parameters for the matter and cosmological constants, respectively. (Again, you need to start with the acceleration equation).

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