COSMOLOGY ASTM108

PROBLEM SET 6

1. It is important to appreciate why it is that the energy densities of different species of relativistic particles in thermal equilibrium all scale as the fourth power of the temperature. Work through the calculation outlined in Section 7.5 of the printed notes entitled 'Particles in Thermal Equilibrium' that leads to the important equation (7.5) for the total degeneracy factor

$$g_* = \sum_{\text{boson}} g_i + \frac{7}{8} \sum_{\text{fermion}} g_i$$

of a mixture of relativistic particle species. (You may assume that the numerical value of the integral in Eq. (7.19) of the notes is as given – the details of how this integral is done is outlined in appendix B and is, of course, non–examinable).

2. Assuming that all neutrino species were in equilibrium at some early epoch in the history of the universe, deduce that the combined energy density of the neutrinos and antineutrino particles was 21/8 times that of the photons (radiation).
After the universe was about 10^{-6} seconds old, it was dominated by the photons, electrons, positrons and neutrinos. From the information given in the lectures and notes, calculate the total energy density of this mixture of particles for a given temperature.

3. Why is it reasonable to ignore the curvature term in the Friedmann equation when talking about the early universe. (Hint: The answer is a consequence of what should have surprised you at the end of Question 1 of Problem Sheet 3).

4. At different times in the history of the early universe, different (relativistic) particle species would have been in equilibrium. The total energy density of all of these particles is given by the sum $\epsilon_{\text{tot}} = \sum_i \epsilon_i$ and it is this that goes into the Friedmann equation:

$$H^2 = \frac{8\pi G}{3} \frac{\epsilon_{\text{tot}}}{c^2}$$

Show that the age of the universe, t, before the epoch of matter–radiation equality is (approximately) given in terms of its temperature, T, by

$$t \approx \frac{10^{20}}{\sqrt{g_*}} \left(\frac{T}{\text{K}}\right)^{-2} \text{ sec}$$

where g_* is the combined degeneracy factor, time is measured in seconds and temperature in Kelvin (K).

5. The electrons become non–relativistic when it becomes energetically unfavourable for photons to convert into electron–positron pairs. Estimate the temperature and age of the universe when this occurs.

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