COSMOLOGY ASTM108

PROBLEM SET 7

1. Work through Sections 7.3 and 8.1 of the printed notes, where it is shown that the universe today should be bathed in a neutrino background, much the same as the cosmic microwave background, but with a slightly lower temperature of 2 K.

2. Given that the present temperature of the neutrino background is actually $T_{\nu,0} = 1.96\,\mathrm{K}$, calculate the present–day energy density of the neutrinos and verify that this is about 0.7 times that of the cosmic microwave background radiation. (This links in with the discussion following Eq. (8.27) of the printed notes. Remember that there are three types of neutrino, each with its own antiparticle and that the degeneracy factor for a neutrino is $g = 1$).

3. Calculate the age and temperature of the universe when its size was comparable to the size of your head.
Calculate the size and age of the universe when its temperature was comparable to that of room temperature.
The highest energy scales that can be attained by earth–based experiments are those at the CERN particle collider, where typical particle energies are of the order of 100 GeV. How old was the universe when particle energies were this high?

5. Suppose that the half–life of the neutron were 94 seconds, instead of 940 seconds. Calculate the mass-fraction of helium–4 in this case once nucleosynthesis has come to an end.
Do you consider nucleosynthesis or decoupling to be a stronger test of the hot big bang theory? Justify your answer.

6. If the Ω–parameter differs from unity during the big bang, it moves away from this value. Can Ω ever become infinite and, if so, what does this correspond to?
Are there any circumstances where Ω may initially have a value less than unity and then evolve so that its value exceeds unity?

7. Some theories of particle physics predict that a class of elementary particle called magnetic monopoles are formed in the early universe when the temperature was $3 \times 10^{28}\,\mathrm{K}$. Assume that these particles are non–relativistic and that the universe is dominated by radiation when they form. Further assume that the curvature term in the Friedmann equation can be neglected at this time ($k = 0$). Calculate the temperature of the universe when the monopoles begin to dominate the dynamics of the expansion if they are created with an initial density that is 10^{-10} times that of the critical density at that time. What are the consequences of your answer for nucleosynthesis?

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