COSMOLOGY ASTM108

PROBLEM SET 8

1 In a radiation–dominated universe, the temperature–time relation is

$$\frac{T}{2\times 10^{10}\,\mathrm{K}} \approx \sqrt{\frac{1\,\mathrm{sec}}{t}}$$

Suppose that the universe is open (negatively curved) and rapidly becomes dominated by its curvature soon after the temperature has fallen to $3\times 10^{25}\,\mathrm{K}$. How is the above relation altered? Calculate the age of the universe in this case when its temperature has fallen to $3\,\mathrm{K}$. [**Hint:** the relationship $aT=$ constant between the temperature and scale factor of the universe always holds].

2 Throughout the course thus far, we have assumed that the neutrinos are massless. However, there is some evidence from particle experiments that they may have a mass, m_ν. Given that the present–day number density of neutrinos, $n_{\nu 0}$, is roughly that of the photons and that this is related to the density of (massive) neutrinos by $\rho_{\nu 0} \approx m_\nu n_{\nu 0}$, derive an expression that relates the mass of the neutrino to the Ω–parameter for the neutrino, $\Omega_\nu \equiv \rho_\nu/\rho_{\mathrm{critical}}$, where ρ_{critical} is the present value of the critical density. Hence, show that

$$m_\nu c^2 = 30\,h^2\Omega_{\nu,0}\,\mathrm{eV}$$

What does the fact that the total density of the universe is near to the critical density tell you about the mass of the neutrino?
Taking the mass–energy of the neutrino to be about $30\,\mathrm{eV}$, estimate the time that such a particle would have become non–relativistic. How old was the universe when this occurred? Is this close to any other important times in the universe's history?
[Note how the above expression relates a particle physics parameter, namely the mass of the particle, directly to the two fundamental observable parameters in cosmology, namely Hubble's constant and the Ω–parameter. By measuring these two quantities, we can employ cosmology to learn something about particle physics].

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